Sediment Transport in Mountain Streams of the Colorado-Wyoming Rocky Mountains: A State of the Art Review

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SEDIMENT TRANSPORT IN MOUNTAIN STREAMS OF THE COLORADO-WYOMING ROCKY MOUNTAINS: A STATE-OF-THE ART REVIEW

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INTRODUCTION

Forest management activities can potentially impact the beneficial uses of mountain streams by affecting the balance between runoff and sediment load which, in turn, can trigger changes in channel morphology. The term "channel maintenance flows" has been defined by the U.S. Forest Service (Troendle, in press) as "the flow regime needed to preserve the integrity of the existing channel and maintain its capacity to transport water and sediment." Instream flows are needed for channel maintenance as well as for maintenance of aquatic habitat and the various functions of riparian zones, but the techniques for defining these instream flows remain unclear for mountain streams (Naiman 1993). Two critical components for understanding channel maintenance flows are incipient motion of particles and sediment routing through mountain stream channels. Understanding linkages between these processes and channel morphology is the key to defining channel maintenance flows and presents one of the greatest challenges for fluvial geomorphology (Rhoads 1992).

The Stage 1a Progress Report for the current project reviewed processes and models of sediment delivery to mountain streams from clearcut logging and timber harvest roads. The Stage 2 Progress Report for the current project examined the effects of clearcut logging on the morphology and substrate of mountain streams. This report completes the project by examining the mechanics of sediment transport after it has entered mountain stream channels and how to link sediment transport with channel morphology. Specific objectives of the paper are to:

1) summarize the basic concepts of sediment transport relations in mountain streams, including incipient particle motion and the scale-linkage concepts of sediment routing and magnitude-frequency analysis.

2) analyze existing sediment data sets collected in streams of the Medicine Bow National Forest.

This paper will be organized in two main sections, as outlined above. First, a review of the literature on sediment transport will be presented, including a summary of approaches prior to 1980, and a more detailed review of major scientific advances since 1980 pertinent to mountain streams. Special attention is given to approaches which were discussed as part of the hearings concerning channel maintenance flows and water diversions in District Court, Water Division 1, State of Colorado. Second, an analysis of sediment relations for existing gaging stations with sediment data in the Medicine Bow National Forest will be presented. This analysis offers an example of how sediment transport relations can be linked with channel morphology in an attempt to better understand channel maintenance flows.

LITERATURE REVIEW

A range of scales of understanding exists in fluvial geomorphology, including, in descending order: drainage basin, hillslopes/channel reach, channel bedforms, channel cross-section, and individual particles. Processes at the scale of a drainage basin and hillslopes were partly reviewed in the Stage 1a Progress Report and will not be considered further here. The following review, therefore, focuses on transport ranging from the scale of the channel reach (sometimes termed sediment routing) to the scale of individual particles (sometimes termed incipient particle motion).

Most of the early work on sediment transport performed by hydraulic engineers using laboratory flume studies with uniform particle sizes or on field studies of alluvial streams beyond the mountain front. Improvements in strategies and instrumentation for sediment sampling have allowed greater collection of field data in mountain streams, especially in the last three decades. Hydrologists and geomorphologists have expanded the scope of sediment transport from one focused on incipient motion to the larger scales. Among the large volumes of research on sediment transport in mountain streams to emerge since 1980, three compilations deserve special mention for advancing the science: <u>Gravel Bed Rivers--Fluvial Processes, Engineering, and Management</u> (Hey et al. 1982), <u>Sediment Budgets and Routing in Forested Drainage Basins</u> (Swanson et al. 1982), and <u>Erosion and Sedimentation in the Pacific Rim</u> (Beschta et al. 1987).

<u>Classification of Solid Sediment in Transport</u>

The solid portion of the total sediment load moves as suspended load, saltation load, and contact load. Suspended load involves relatively small particles, typically clays and silts, which are kept suspended in the water column by the upward components of turbulence. Saltation load involves intermediate size particles, typically sands, which move in short jumps, alternately in brief suspension and at rest on the channel bed. Contact load involves relatively large particles, typically gravel and larger, which move by rolling and sliding along the channel bed (Vanoni 1975).

Shen and Li (1986) partition the solid load into wash load and bed material load based on the supply rate and transport capability of various sized particles under a particular flow regime (Figure 1). The transport capability, in turn, will be a function of the flow capacity (the discharge of the stream) and the flow competence, or the strength of the flow, as measured by velocity, shear stress, or stream power. Sizes for which the rate of supply is exceeded by the transport capability (supply limited) comprise the wash load. Sizes for which the transport rate is exceeded by the supply rate (energy-limited conditions) comprise the bed

material load. Although the division between wash load and bed material load will vary depending on sediment supply rate and flow transport capability, 0.0625mm is the size used by some researchers to divide the solid load.

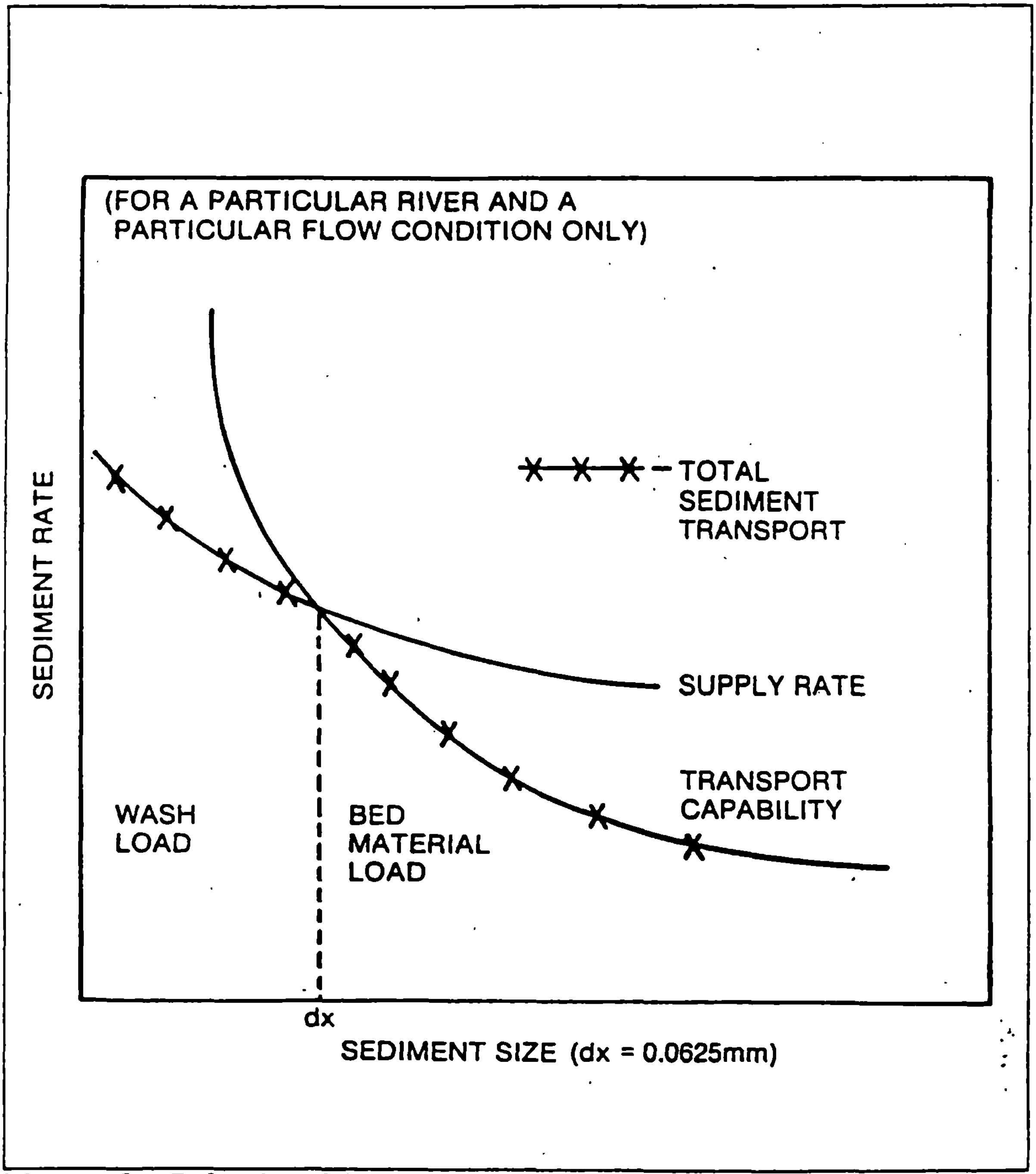

Figure 1. Relationships of sediment transport rate to supply rate and transport capability (Shen and Li 1976).

The division of solid load into suspended load, saltation load, and contact load...or between wash load and bed material load...will vary over time as sediment supply and strength of flow vary. Moreover, some particles may move by all modes of transport at a given flow condition. Therefore, a practical method for partitioning the solid load is based on the instruments used to sample the sediment. For instance, suspended sediment is defined by Guy and Norman (1970) as the portion measured using depth-integrated samplers (e.g., DH-48, DS-49, DS-59). This procedure, utilizing the equal transit rate method, requires a continuous sample to within three inches of the stream bed (Rosgen 1980). The portion of the solid load moving within three inches of the bed as measured with a Helley-Smith pressure differential sampler. Particles which saltate more than three inches above the bed may be measured as suspended load at the point of sampling, while the same sized particles may also be measured as bedload at the same sampling station. Although it is beyond the scope of this paper to review sediment sampling techniques, clearly the strategy of sampling and the sampling efficiency of any given device will influence the distinction made between modes of transport and the total amount measured.

Properties of Sediment Affecting Transport

The properties of sediment affecting transport rate include properties of individual particles and of the mix of particles in the substrate.

Size and Specific Gravity

The most important sediment property is size. A number of scales of sediment size exist, but the most widely accepted scale is that of the American Geophysical Union (Table 1)(Lane 1947). In an analysis of forces acting on an individual particle, however, it is the specific gravity of the particle which is more important than the size, and the specific gravity of the sediment is the ratio of the specific weight (mass/volume) of the sediment divided by the specific weight of the water. Specific weight will vary with lithology of the sediment, but a value of 2.65 (from that for quartz sand) is often assumed. Carbonate rocks, in contrast, typically have specific weights of 2.85. In some calculations, the specific weight of a sediment deposit is considered, and this value will be less than that for the individual clasts because of voids in the deposit. The specific weight of a deposits will depend on it composition, environment of formation, and time since deposition (Vanoni 1975).

Shape and roundness

Shape refers to the form of the particle without reference to the sharpness of its edges, while roundness depends on the

Table I. Classification of sediments by size (Lane 1947).

	Size range[1]				Approximate sieve mesh openings per inch	
	Millimeters					
Class name			Microns	Inches	Tyler screens	United States standard
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Very large boulders		4,096-2,048		160-80		
Large boulders		2,048-1,024		80-40		
Medium boulders		1,024-512		40-20		
Small boulders		*512-256		20-10		
Large cobbles		256-128		10-5		
Small cobbles		*128-64		5-2.5		
Very course gravel		64-32		2.5-1.3		
Course gravel		*32-16		1.3-0.6		
Medium gravel		16-8		0.6-0.3	2-1/2	
Fine gravel		8-4		0.3-0.16	5	5
Very fine gravel		*4-2		0.16-0.08	9	10
Very course sand	2-1	2.000-1.000	2,000-1,000		16	18
Course sand	1-1/2	*1.000-0.500	1,000-500		32	35
Medium sand	1/2-1/4	0.500-0.250	500-250		60	60
Fine sand	1/4-1/8	0.250-0.125	250-125		115	120
Very fine sand	1/8-1/16	*0.125-0.062	125-62		250	230
Course silt	1/16-1/32	0.062-0.031	62-31		270	
Medium silt	1/32-1/64	0.031-0.016	31-16			
Fine silt	1/64-1/128	0.016-0.008	16-8			
Very fine silt	1/128-1/256	0.008-0.004	8-4			
Coarse clay	1/256-1/512	0.004-0.0020	4-2			
Medium clay	1/512-1/1,024	0.0020-0.0010	2-1			
Fine clay	1/1,024-1/2,048	0.0010-0.0005	1-0.5			
Very fine clay	1/2,048-1/4,096	0.0005-0.00024	0.5-0.24			

[1]Reprinted with permission from *Sedimentation Engineering*, edited by Vito A. Vanoni and published by American Society of Civil Engineers, New York.

[2]Recommended sieve sizes are indicated by an asterisk (*).

sharpness of the edges. Vanoni (1975) reviews the techniques for assessing the effect of shape on transport, but the effect is rarely incorporated into equations estimating thresholds or rates of transport.

Size Frequency Distribution

Sediment samples, whether acquired from the channel directly or from transport measuring devices, are usually separated into size classes and then described as a cumulative size-frequency curve on log-probability graph paper. The probability scale is labelled "percent finer" so that the d84 is the size for which 84 percent of the sample is finer. The geometric standard deviation

of the sample, used to measure the sorting, can be calculated by a number of formulae. If the distribution of sizes is log-normal, then (Otto 1939):

$$\text{sorting} = (d84/d16)^{0.5} \tag{1}$$

$$\text{mean } d = (d84 \times d16)^{0.5} \tag{2}$$

With a mix of particle sizes, small particles may be hidden by larger particles. Parker and Klingeman (1982) and Andrews (1983) have shown that the small particles begin to move at nearly the same time as the larger particles, a condition of near equal mobility which was not recognized by Shields. In the case of low submergence, the mobility of smaller particles in the mix may be enhanced as flow is concentrated around larger boulders (Mizuyama 1977). Where channel bed topography is significant, some but not all particles of a given size may move when a threshold is exceeded since some particles are protected in deep pockets (Andrews and Erman 1986, Andrews 1990). Reid and Frostick (1984) found that in the absence of high flows, fine sized particles move through the interstices of the bed and strengthen the bed "very much like mortar in a stone wall." This effect can increase the flow threshold needed to initiate particle motion by a factor up to five times.

Properties of Flow Affecting Sediment Transport

Water flowing over a bed of sediment exerts forces of drag, lift and buoyancy on grains that tend to move or "entrain" them (Figure 2). The lift force is caused by a velocity difference between the bottom and top of the grain which creates a vertical pressure gradient. Although rarely measured, it can be as important as the drag force in entraining particles, especially in sand-bed streams (Knighton 1984). For sediment of sand size or coarser, the resisting forces are caused mainly by the weight. Silts and clays resist entrainment more because of cohesion than because of size. Several properties of flow affect sediment transport: kinematic viscosity, density, temperature, and wash load concentration. However, they will not be discussed here because of their low relative importance in mountain streams.

Thresholds of Incipient Particle Motion

The first studies of incipient particle motion examined critical values of velocity, including the often-cited Hjulstrom (1935) curve (Figure 3). However, two streams with the same mean velocity and different depths will produce different rates of transport. Therefore, shear stress, also termed tractive force is the preferred variable. An estimate of boundary shear stress on the channel bed can be obtained with the product of specific weight

of water, hydraulic radius (a surrogate variable for water depth), and channel gradient (Knighton 1984). An analysis of forces (Figure 2) on a spherical grain of diameter D on a flat bed, equating the applied forces with the resisting forces, yields the more precise equation for shear stress shown. Shields (1936) recognized that the critical shear stress depended not only on particle size but also on bed roughness. The Shields diagram relates a dimensionless critical shear stress to a particle Reynolds number which defines the bed roughness condition (Figure 4). Shields determined that the critical dimensionless shear stress had a constant value of 0.060 when the particle Reynolds number exceeded 500. Unfortunately, Shields did not present the experimental data from which he derived his famous diagram. Wiberg and Smith (1987) demonstrate how Shield's technique is inappropriate for steep mountain streams where the substrate is poorly sorted and relative roughness is high. Shields himself did not intend for his model to be used for estimating bed material transport in natural rivers but only as a laboratory tool for understanding the laws governing behavior of materials (Shulits and Hill 1968).

Knighton (1984) has summarized the limitations to these approaches:

> They fail to cope adequately with the variability of either flow conditions near the stream bed or bed material characteristics. Short-term pulsations in the flow can give rise to instantaneous stresses of at least three times the average, so that particles may be entrained at stresses much lower than predicted. Sediment entrainment is a function not only of the average shear stress on the bed but also of the intensity of turbulence above it, which can exert an impulse force on grains. Since eddy size and hence the energy available for moving grains are related to the size of the system, the size of the channel can influence the entrainment process (Raudkivi 1976). Natural bed material is neither spherical nor of uniform size. Larger particles may shield smaller ones from direct impact so that the latter fail to move until higher stresses are attained. Characteristics other than size influence particle mobility, notably the degree of grain exposure (Fenton and Abbott 1977), bed relief and sediment fabric (Laronne and Carson 1976). Thus for a given grain size > 8 mm, the Shields' threshold criterion may vary by nearly an order of magnitude depending on whether the bed is loosely or tightly packed (Church 1978).

In mountain streams, the steep gradient can affect transport rate by causing local supercritical flow and hydraulic jumps in the vicinity of large boulders and steps. At a given flow, the critical ratio of depth-to-particle size, D/d, is less for mountain streams (Lisle 1987). Regarding the critical dimensionless shear stress introduced by Shields (1936), many subsequent laboratory

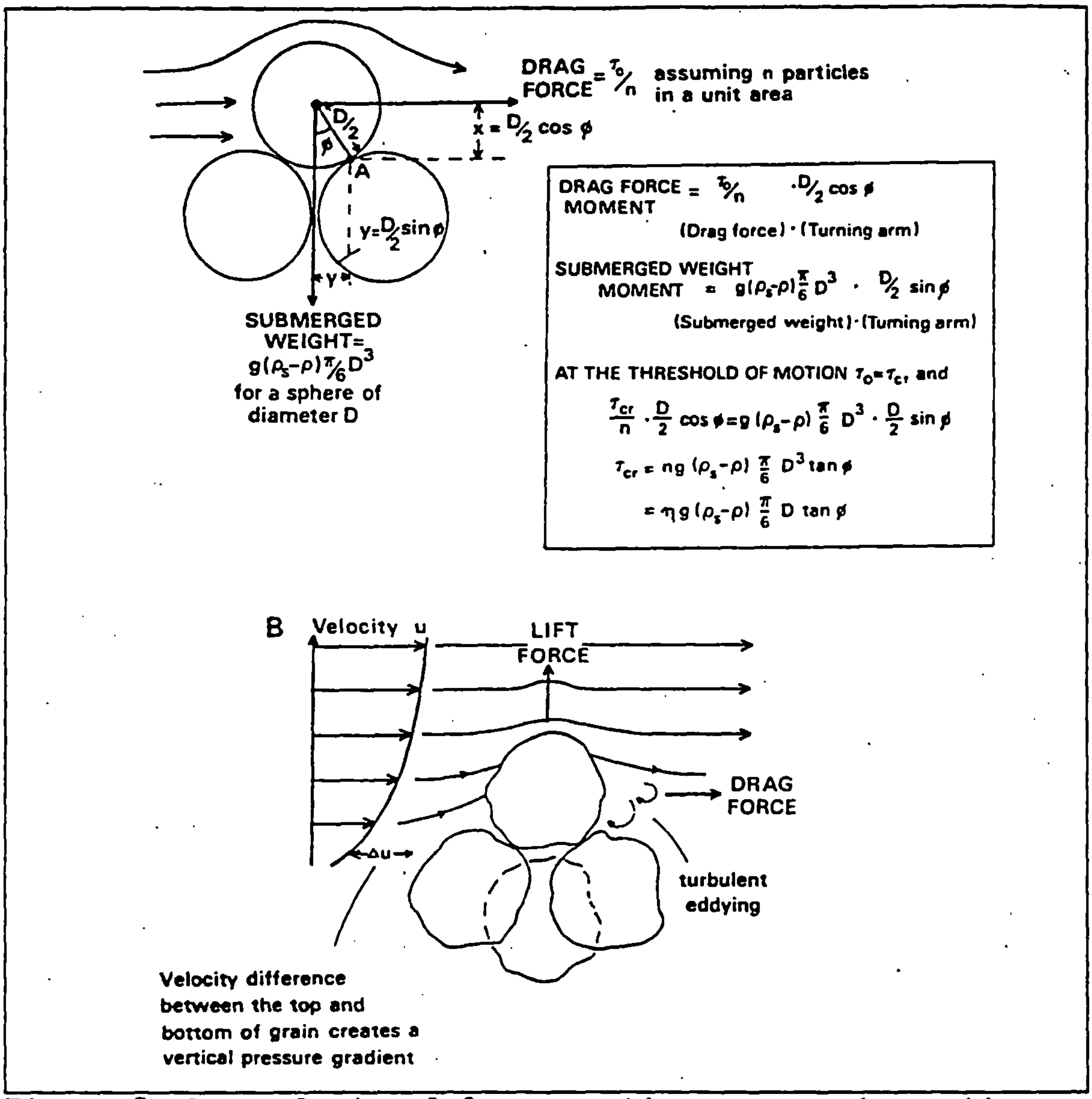

Figure 2. An analysis of forces acting on a grain resting on a horizontal bed (Knighton 1984).

studies have shown values converging at 0.030, or half of what Shields reported. Studies in natural rivers with coarse substrates have found a wide range of critical dimensionless shear stress, ranging from 0.010 to 0.25, with a median value of 0.045 to 0.060 (Andrews 1983). Andrews found that the critical dimensionless shear stress can be expressed as a function of the ratio between particle size and the d50 of the sample. Andrews showed that critical dimensionless shear stress varies almost inversely proportional to the particle diameter for a nonuniform substrate. Therefore, bed particles between 0.3 and 4.2 times the d50 are entrained at nearly the same discharge. In another study of gravel-bed streams in the Colorado Rockies, Andrews (1984) found

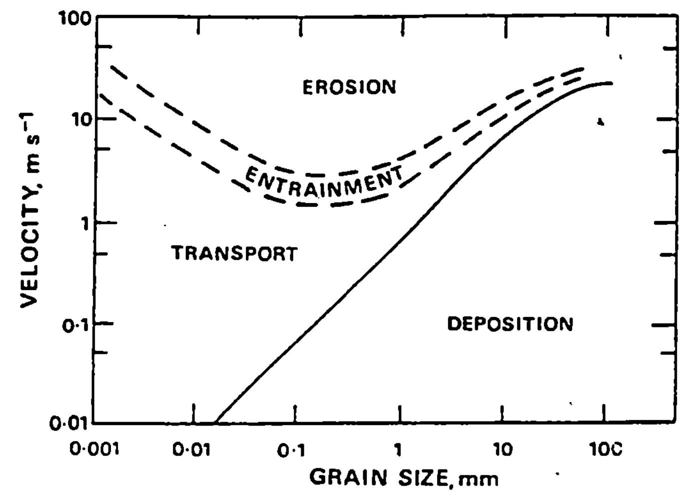

Figure 3. The Hjulstrom (1935) diagram for thresholds of erosion and deposition (Knighton 1984).

that the critical dimensionless shear stress at bankfull stage was 0.046 which was exceeded an average of 8.1 days per year. The critical dimensionless shear stress for the 100-year flood was only 0.068. A progressive increase in the portion of bed material in transport is observed as critical dimensionless shear stress increases from 0.20 to 0.60 (Andrews 1992). From these data, Andrews concludes that transport of bed material is relatively frequent, but large bed material transport events are rare. Finally, estimates of velocity, shear stress, or stream power at the bed of steep mountain channels with large relative roughness is difficult at best. Many equations developed for these streams have not proven useful when tested elsewhere under similar conditions (Thorne 1985).

Rates of Sediment Transport

The rate of wash load transport depends more on sediment supply than on transport capability of the stream (Figure 1). However, a portion of the supply of fine-sized sediment emanates from the channel bed beneath an armor layer of coarse sized particles. Therefore, a two-phased regime of wash load transport may exist, separated by the threshold of incipient motion for the armor layer on the bed protecting fines.

The concentration of sediment as wash load or suspended load is typically expressed as a function of water discharge in a curve known as a suspended sediment rating curve. The suspended sediment concentration, Q_{sus}, is related to discharge, Q, in a power function (Figure 5A):

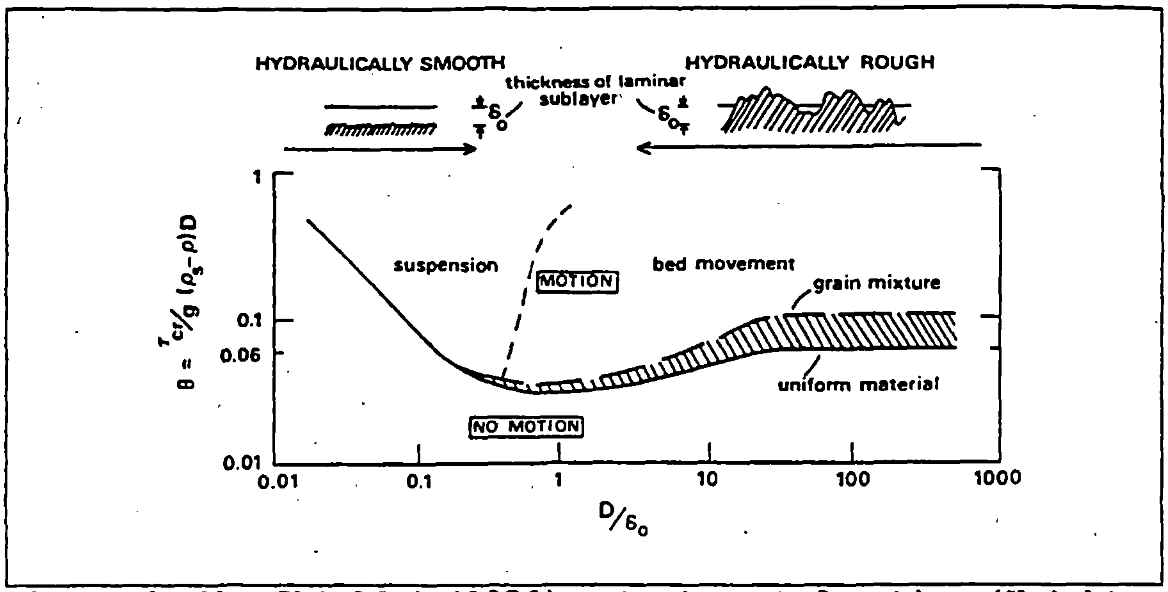

Figure 4. The Shields' (1936) entrainment function (Knighton 1984).

$$Q_{sus} = rQ^j \tag{6}$$

A large amount of scatter is common in suspended sediment rating curves for several reasons. First, the wash load sediment supply is highly variable over time. A hysteresis effect may exist where larger loads are transported on the rising limb of storm-period and/or annual hydrographs for a drainage before the supply has not been flushed. Fine sized sediment which was deposited during the waning stages of previous high flows is released early during the rising limb of subsequent events (Lisle and Hilton 1992). Second, additional scatter can be attributed to the combination of wash load and suspended load. The former depends on supply but the latter includes wash load and a portion of the saltating load. Third, the rating curve may have sharp breaks or be a polynomial function (Singh et al. 1986). Fourth, suspended sediment rating curves are sometimes compiled with instantaneous sediment samples coupled with mean daily flows, thereby missing diurnal fluctuations (Singh et al. 1986) and storm-period lags between peak flows and peak sediment concentrations (Knighton 1984). Despite the scatter, suspended sediment rating curves have been used with some success to document impacts of forestry activities (Flaxman 1975, Rosgen 1975, Sundeen 1977).

The rate of bed material transport depends more on the transport capability than on the sediment supply (Figure 1). Many equations to estimate bed material transport relate the rate of bed material transport per unit channel width, q_{bed}, to discharge, shear stress, or stream power per unit width in excess of a threshold value, respectively q_t, t_t, p_t. The stream power per unit width (also termed unit stream power) is the the product of specific

weight of water, water discharge, and gradient (expressed as a decimal fraction), all divided by channel width.

$$q_{bed} = a(q-q_t) \tag{7}$$

$$q_{bed} = b(t-t_t) \tag{8}$$

$$q_{bed} = b(p-p_t) \tag{9}$$

Bagnold (1977) refined equation 9 to account for relative roughness (Figure 5B) and Leopold and Emmett (1976) developed a suite of curves using Bagnold's (1966) equation for different particle sizes (Figure 6):

$$q_{bed} = (p-p_t)^{3/2}d^{-2/3}D50^{-1/2} \tag{10}$$

Some authors have incorrectly stated that equation 10 implies that bed material transport rate increases as flow depth decreases. However, recall that flow depth is also part of the unit stream power term, p, so that q_{bed} is actually proportional to $D^{1/3}$. Bedload transport rating curves are subject to significant scatter, due to factors such as break-up of the armor layer, bimodal particle size distributions, changes in sediment storage in pools and behind large woody debris (Klingemann and Emmett 1982, Emmett et al. 1983, Estep and Beschta 1985, Ketcheson 1986, Duncan et al. 1987, Sidle 1988, Nakamura and Swanson 1993). Because the threshold value of stream power is difficult to determine,Rosgen (1980) suggests developing a suite of curves relating bedload transport rates to stream power (not excess stream power) for a suite of D50 values at each channel reach. Moreover, he suggests developing different bed material transport curves for reaches of contrasting stability as judged by the Pfankuch (1975) semi-quantitative evaluation scheme. Alternatively, separate sediment rating curves could be developed for different reach types: meandering, braided, pool-riffle, step-pool, cascade. The notion of developing separate sediment rating curves based on reach characteristics deserves attention, whether for bed material load or for wash load, but the value of the approach has not been confirmed by data. Based on laboratory experimental data, Young and Davies (1991) postulate that channel morphology is adjusted to discharge in a manner which maximizes rates of bed material load transport. Hoey (1992) proposes that various scales of bedforms in gravel-bed streams be used to classify pulses and waves of bed material transport. Lekach and Schick (1992) are less optimistic that channel morphology can be used to characterize bedload pulses. Grant (1992) has shown that steps (from rocks, riffles, boulders, large woody debris) develop under conditions of gradients exceeding two percent, widely-sorted sediment, and low sediment supply. Whittaker (1987) was able to model sediment transport through a laboratory simulation of a step-pool system.

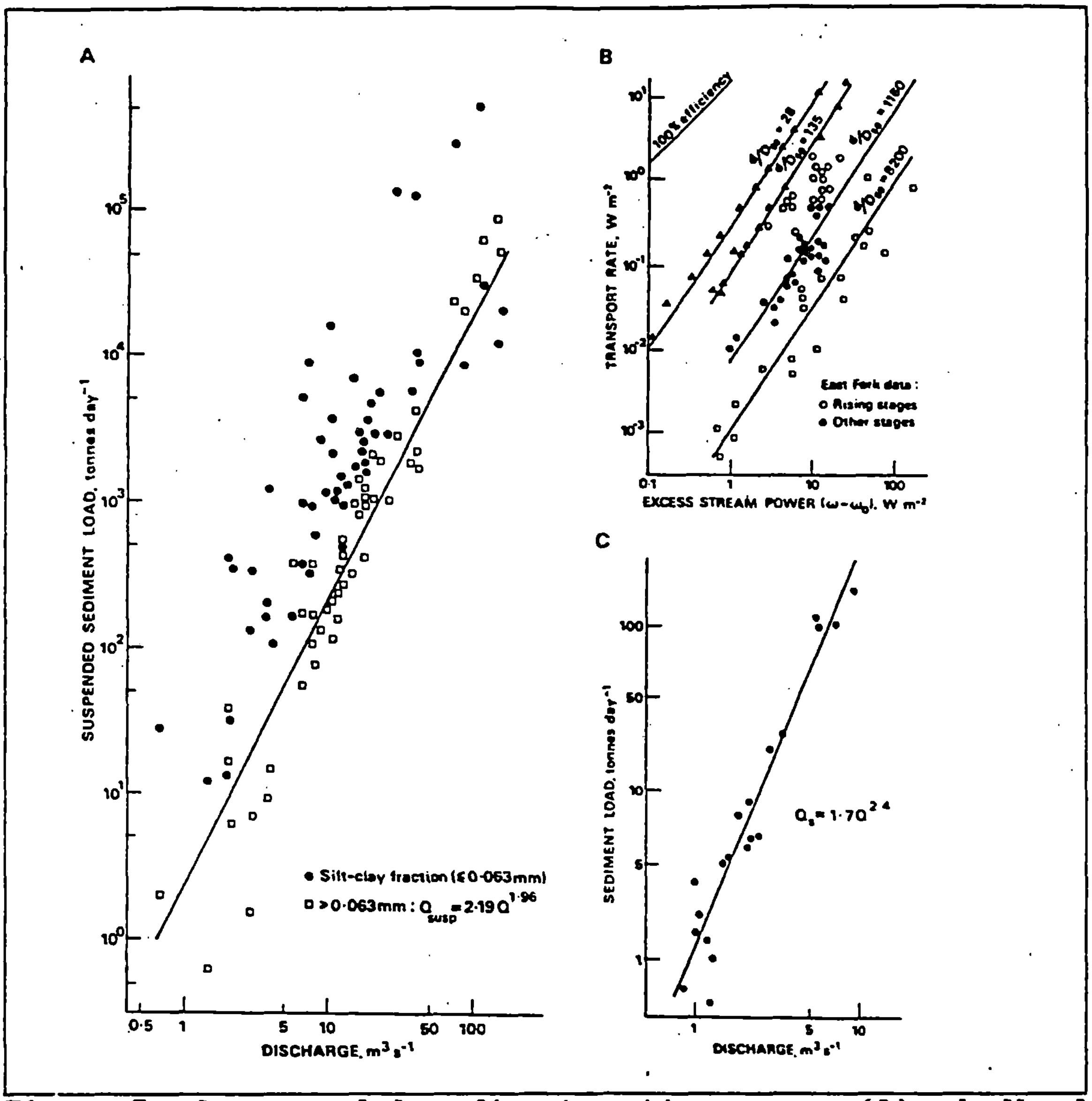

Figure 5. A suspended sediment rating curve (A), bedload transport curve (B), and total sediment load curve for three different rivers (Knighton 1984).

Ned Andrews and Jonathan Nelson of the U.S. Geological Survey Water Resources Division in Boulder, Colorado, are close to completing a method for calculating flow and bed material transport in streams with poorly sorted gravel substrate and large relative roughness, d/D. The procedure will allow estimates of bed material transport at, above, and below bankfull stage using field observations of channel morphology and substrate. Wilcock (1992) cautions that back-calculations of critical shear stress using the largest grain found in samples of transported sediment are subject to large errors. It is preferable to use the d50 to define the flow competence of the sediment mix.

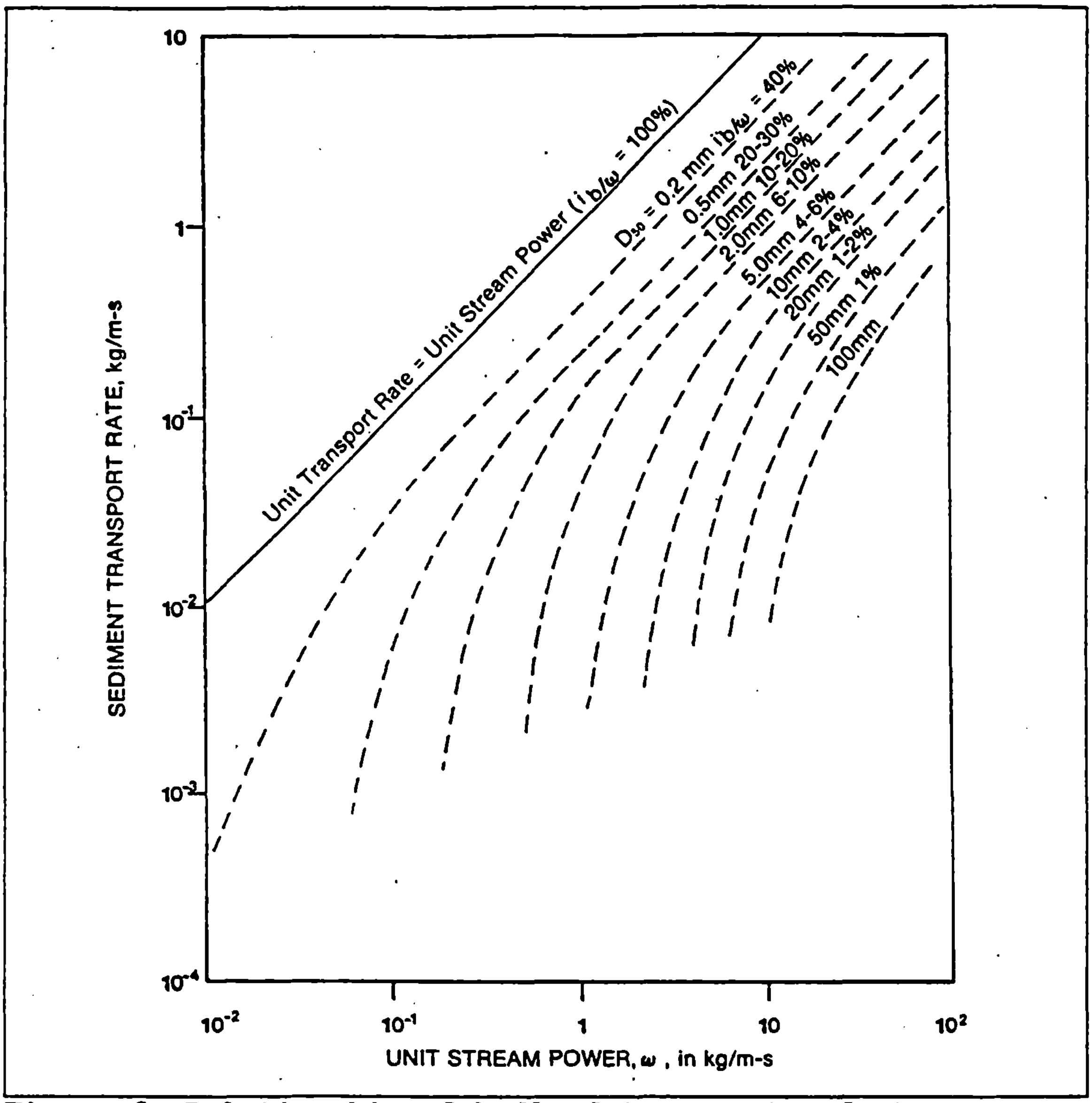

Figure 6. Relationship of bedload transport and stream power for the East Fork River, Wyoming (Leopold and Emmett 1976).

Many other bed material transport formulae exist, with thorough reviews by Shulits and Hill (1968) and Vanoni (1975). Two merit special attention. Meyer-Peter and Muller (1948) developed a formula for estimating bed material discharge under conditions of steady flow:

$$q_{bed} = [(39.25)(q^{2/3})(s)] - [(9.95)(d50)] \qquad (11)$$

This equation as stated above is only valid for the foot-pound-seconds units of measurements and is similar in form to another oft-cited bed material transport formula devised by Schoklitsch

(1934). The bedload cannot be computed by summation of various size fractions. It was developed for streams with well-sorted substrates ranging in median size from 3.1 to 28.6 mm and lacking in bedforms (Vanoni 1975). Equation 11 assumes that sediment supply is not limited, an erroneous assumption for bed material load (Figure 1) and one which results in overestimating rates of bed material transport (Simpson 1978). A study by Smart (1984) demonstrated that the Meyer-Peter and Mueller equation (equation 11) seriously underestimates sediment transport rates in channels steeper than three percent, mainly because of deficiencies in the form resistance factor. Hans Einstein (1950), the son of Albert, developed a method for estimating bed material discharge for individual size fractions of the bedload, following 17 steps outlined by Vanoni (1975). The complex procedure involves field measurements of channel cross-section dimensions, determination of sediment size-frequency distribution, and hydraulic calculations. The method was developed from laboratory flume experiments using fine graded sand. White and Day (1982) found promising results when modeling bedload transport for individual size fractions. Moreover, equations 7-11 all presume that bedload transport rates rise progressively as flow strength increases. In fact, bedforms (Brayshaw 1985, Hassan et al. 1992), changing integrity of the armor layer, pebble clusters, and kinematic waves of sediment (Reid and Frostick 1984) are several reasons why bedload transport rates can be "out-of-phase" with changing hydraulic conditions.

Scale-Linkage Concepts

Sediment transport relations, by themselves, do not reveal impacts on channel morphology. Two concepts which link sediment transport with channel morphology are "sediment routing" and "magnitude-frequency analysis."

Sediment Routing

Sediment routing refers to sediment movement through the channel network. In mountain streams, the movement is often discontinuous due to concentrated inputs, highly variable transport capabilities of the stream over space and time, and opportunities for in-channel storage. The basic ingredients of sediment routing studies are (Swanson et al. 1982): "identification of storage sites, transport processes, and linkages among them, and the quantification of storage volumes, and rates of transport processes."

Sites for sediment storage in mountain streams include (Ashley 1990): floodplains and terraces, the active channel bed at low flow, channel forms (point bars, alternate bars, transverse bars), solidary bars (tributary bars, slackwater bars), bedforms (dunes, ripples) and steps...each with a different residence time (or

turnover time) of the sediment. A review of the processes by which sediment is transferred between these "reservoirs" is beyond the scope of this paper, but has been the subject of numerous studies. Residence time has been defined by Dietrich and Dunne (1978) as the volume of sediment stored per unit channel length, V_s (cubic meters/meter), divided by the bedload discharge rate, Q_{bed} (cubic meters/year). If V_s and Q_{bed} can each be defined as a power function of drainage area, A, then the residence time, R_t (years), can also be defined on the basis of drainage area:

$$V_s = aA^m \quad (12)$$

$$Q_{bed} = bA^n \quad (13)$$

$$R_t = V_s/Q_{bed} = aA^m/bA^n = (a/b)A^{m-n} \quad (14)$$

Successful applications of this approach have been published by Dietrich and Dunne (1978), Madej (1984, 1987), Kelsey et al. (1986), and Marston (1993).

For the purpose of understanding channel maintenance flows, it seems appropriate to use the channel reach as the primary landscape unit for sediment routing studies. The key is to delineate channel reaches using those variables which sediment transport studies have identified as critical: the mix of particle sizes in the substrate, measures of relative roughness, bed topography, channel planview pattern, channel gradient, channel cross-section shape, presence of hydraulic jumps, sediment storage effects of large woody debris, and whether the channel is supply-limited or transport-limited. Systems of channel reach classification are being devised by Rosgen (private consultant in Durango, Colorado), Dave Montgomery (University of Washington), Gordon Grant (USFS in Corvallis, Oregon) and others which could potentially be applied in the manner being described. Sediment rating curves could then be developed for reaches rather than for single cross-sections alone.

Magnitude-Frequency Analysis

Magnitude-frequency analysis is a technique devised by Wolman and Miller (1960) to identify the discharge that moves the greatest volume of sediment. The technique can be explained graphically (Figure 7). First, a plot of flow frequency is obtained for a given station on the stream, for example in units of number of days per 500 years. A sediment rating curve is then compiled, with units of transport in tons per day. The product of flow frequency and sediment discharge produces a third curve, total sediment transport in units of tons per 500 years. The peak of the total transport curve is the "effective" (or "dominant") discharge...the discharge that transports the most sediment when accounting for both frequency and transport capability of each flow. For the small, humid temperate region streams studied by Wolman and Miller

with subdued relief and good vegetation cover, the effective discharge occurred once every 1-2 years, a quite frequent event. Baker (1977) showed how changing the threshold of transport could alter the effective discharge (Figure 8), and he found that higher magnitude events can be the effective discharge if the substrate is resistant. Similarly, the effective discharge will contrast in streams with different skewness or kurtosis of the flow frequency curve or the slope of the transport curve.

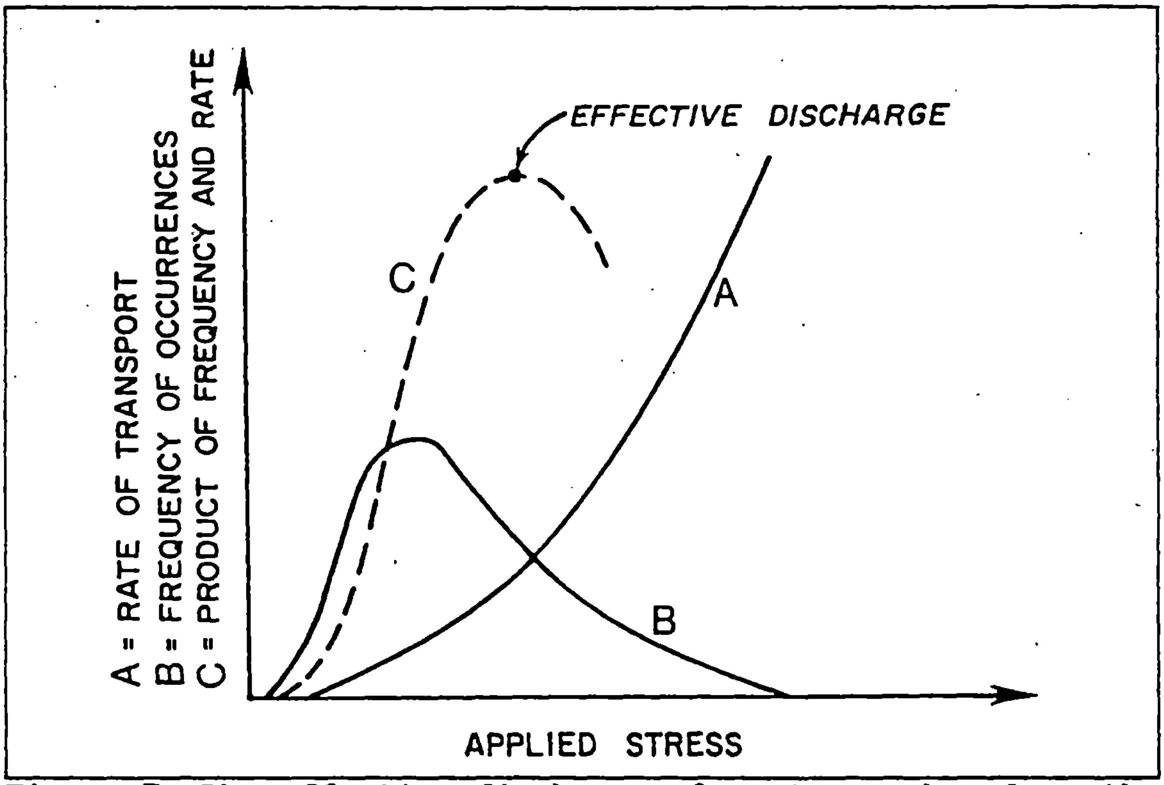

Figure 7. The effective discharge of a stream, based on the product of flow frequency and rate of sediment transport (Wolman and Miller 1960).

Andrews (1980) compared the effective discharge with the bankfull discharge to determine whether flows that account for the greatest mass transport (suspended load and bedload) over time might also be the channel forming flows for 15 gaging stations in the Yampa River basin of Colorado and Wyoming. The recurrence interval for the effective discharge ranged from 1.18 to 3.26 years using the annual flood series. At all gaging stations, the effective discharge and the bankfull discharge were nearly equal, indicating that the channels were adjusted to their effective discharge. Using just suspended sediment, Webb and Walling (1988)

found an equality between effective discharge and bankfull discharge, but these were events of moderate magnitude and frequency. Wesche (1989) found effective discharge (using suspended sediment only) in the Big Sandstone Creek drainage of the Sierra Madre of Wyoming to be less than 10 times the average annual flow and less than bankfull discharge.

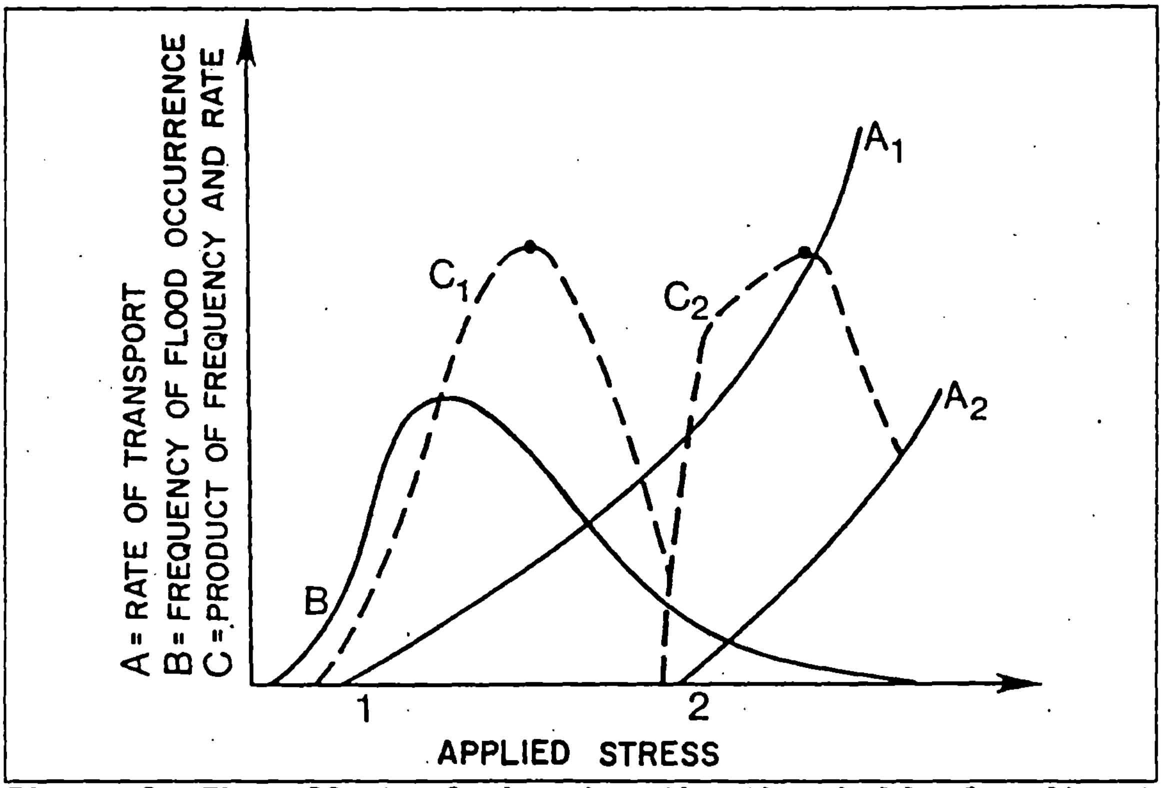

Figure 8. The effect of changing the threshold of sediment transport on the effective discharge (Baker 1977).

Others (Pickup and Rieger 1979, Carling 1988) point out that the inequality of bankfull discharge and effective discharge indicates that channel morphology reflects a "memory" of past catastrophic events. Such channels would not have achieved a condition close to steady state and the effective discharge would not be the channel forming discharge.

These studies indicate that channel maintenance flows are being received if the effective discharge is equated with the bankfull discharge. If the effective discharge is greater than the bankfull discharge, the flows which transport the greatest mass of sediment over time cannot be contained in the channel; channel capacity is insufficient; these streams are transport-limited ("energy-limited"). If the effective discharge is significantly less than the bankfull discharge, then the channel is degrading;

these streams are "supply-limited." Of course, the implementation of the Wolman-Miller approach depends on accurate compilation of flow frequency and sediment rating curves. At present, this approach is restricted to gaging station sites, although techniques do exist for potentially extrapolating flow frequency curves and sediment transport curves to ungaged sites.

ANALYSES OF DATA FROM THE MEDICINE BOW NATIONAL FOREST

Only seven gaging stations exist in the Medicine Bow Mountains with water and sediment discharge records over a period of years:

1) on the west slope of the Sierra Madre, draining into the Colorado River system via the Little Snake River:

    a) East Fork Savery Creek,
    b) Battle Creek,
    c) Big Sandstone Creek, and
    d) North Fork Little Snake River.

2) on the east slope of the Sierra Madre, draining into the North Platte River system:

    e) Coon Creek and
    f) East Fork Encampment River.

3) on the east slope of the Medicine Bow Mountains, draining into the North Platte River system via the Laramie River:

    g) Nash Fork Little Laramie River.

These watersheds range in elevation from 6,500 to 12,000 feet, with local relief between 500-1500 feet, and vegetation cover of lodgepole pine and aspen. The drainages are moderately dissected and deeply incised. The annual hydrograph is dominated by snowmelt with peaks typically occurring during June and base flow from October through March.

Magnitude-frequency analyses were performed on these seven gaging stations (Figures 9-15). The flow frequency curves were derived from multiple regression equations which use drainage area and mean basin elevation to estimate flow events in mountain stream of Wyoming with recurrence intervals between 2 and 500 years (Lowham 1988). The flow frequency data are labelled in number of days the event would occur in a 500-year period. For instance, the 25-year peak flow would occur 25 times in a 500-year period. The bedload transport curves, in units of tons per day, were derived from unpublished data collected with a Helley-Smith sampler by the U.S. Geological Survey (gages a-c), U.S. Forest Service (gages e-f), and Wyoming Water Research Center (gages d and g). Only the bedload was analyzed because the channel is formed in course materials. Bankfull discharge was estimated in several stages. First, the bankfull stage was estimated using the method of Wolman (1955). It is the height above the thalweg at which the width-depth ratio becomes a minimum. The velocity at bankfull was calculated using the Manning equation; the method of Jarrett (1984) was used to estimate Manning's "n" in the equation.

The effective discharge had a recurrence interval which ranged from 2 to 5 years. In every case, the effective discharge is greater than the bankfull discharge. The ratio of effective discharge-to-bankfull discharge ranges from 2.01 (Nash Fork) to 7.03 (Battle Creek); it provides a measure of the degree to which channel morphology is not adjusted to the prevailing regime of water and sediment discharge. Sensitivity analysis can be used to show that reasonable errors in estimating flow frequency or sediment transport rates do have a great affect on the value of effective discharge. The channels have an insufficient capacity to transport the flows which account for the greatest mass transport of sediment over time. The channels reflect a memory of past events, including possibly mass wasting, glaciation, and impacts from mining and tie drives. In the case of Nash Fork, river terraces of fluvial-glacial deposits provide an abundant supply of sediment to the channel.

Increasing diversions of low flows from transport-limited of the Medicine Bow National Forest would not greatly affect the inequality of effective discharge and bankfull discharge. Flows close to the effective discharge should be maintained to provide transport of sediment and maintain the channel. It can be argued that the channel morphology has not adjusted to the prevailing regime of water and sediment, so diverting water would not disrupt equilibrium conditions.

Table II. Bankfull & effective discharge for gaging stations.

Drainage Basin	Ac	R	S	n	Qbf/RI	Qef/RI	Ad	ELm
East Fork Savery Cr.	12.1	.647	.003	.046	16.1/<2	105/4	5.88	8,978
Battle Creek	18.2	.740	.013	.079	32.0/<2	225/3	13.0	9,671
Big Sandstone Creek	14.4	.661	.013	.080	23.2/<2	90/3.5	9.81	7,562
N. Fk. Little Snake R.	21.0	.758	.063	.143	45.5/<2	170/3.5	9.59	9,709
Coon Creek	23.6	.926	.024	.095	55.7/<2	120/3.5	6.53	9,535
Nash Fk. L. Laramie R.	25.0	1.16	.080	.146	79.7/<2	160/4	7.37	10,405
East Fk. Encampment R.	8.0	.690	.027	.105	14.5/<2	72/4	3.53	9,586

Ac - Channel cross-section area at bankfull stage (square feet)
R - Hydraulic radius at bankfull stage (feet)
S - Channel gradient at bankfull stage (feet/feet)
n - Manning's roughness coefficient (estimated by Jarrett method)
Qbf - Bankfull discharge (cubic feet per second)/Recurrence interval (years)
Qef - Effective discharge (cubic feet per second)/Recurrence interval (years)
Ad - Drainage area (square miles)
ELm - Area-weighted mean elevation of the drainage (feet)

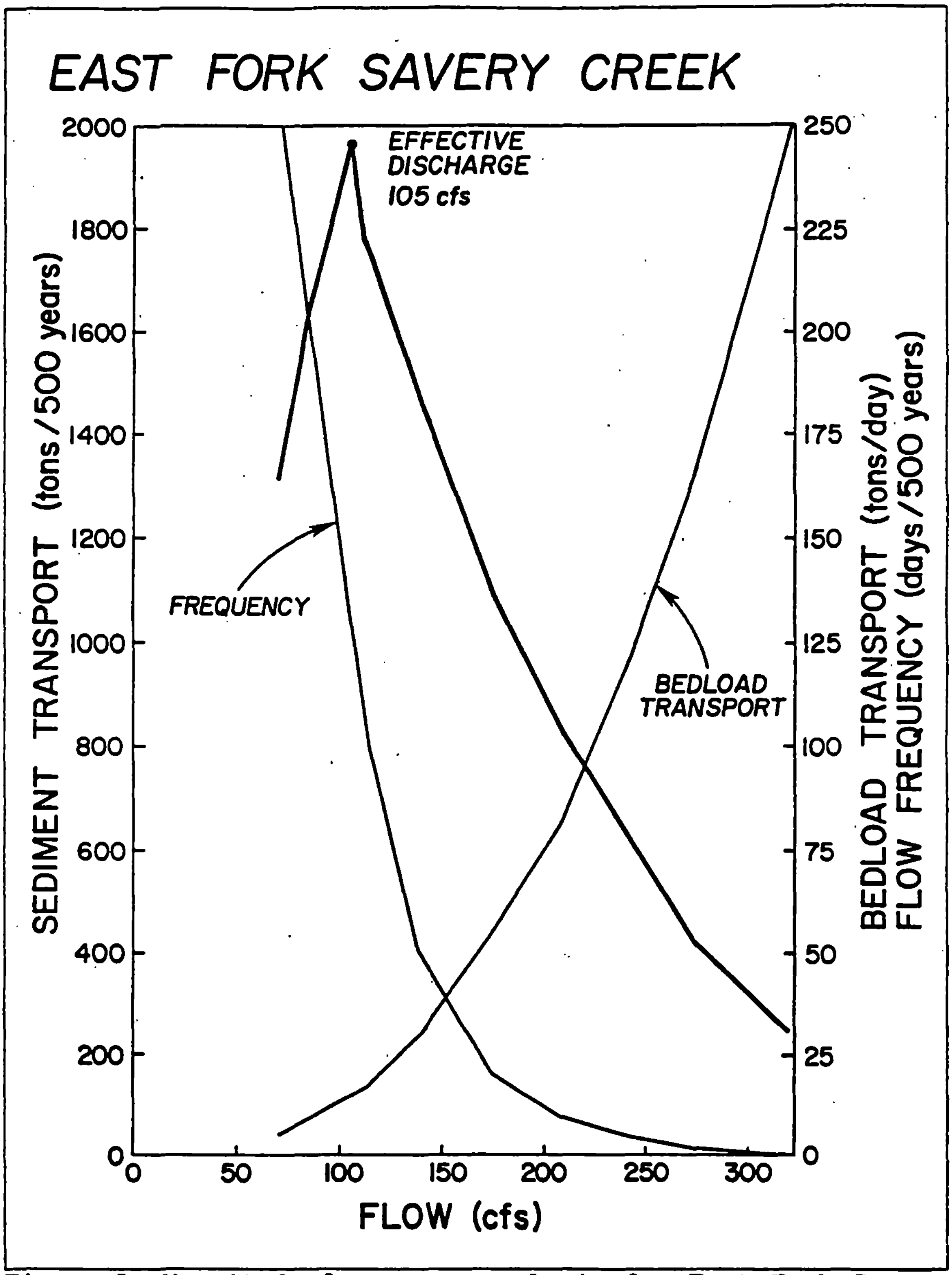

Figure 9. Magnitude-frequency analysis for East Fork Savery Creek.

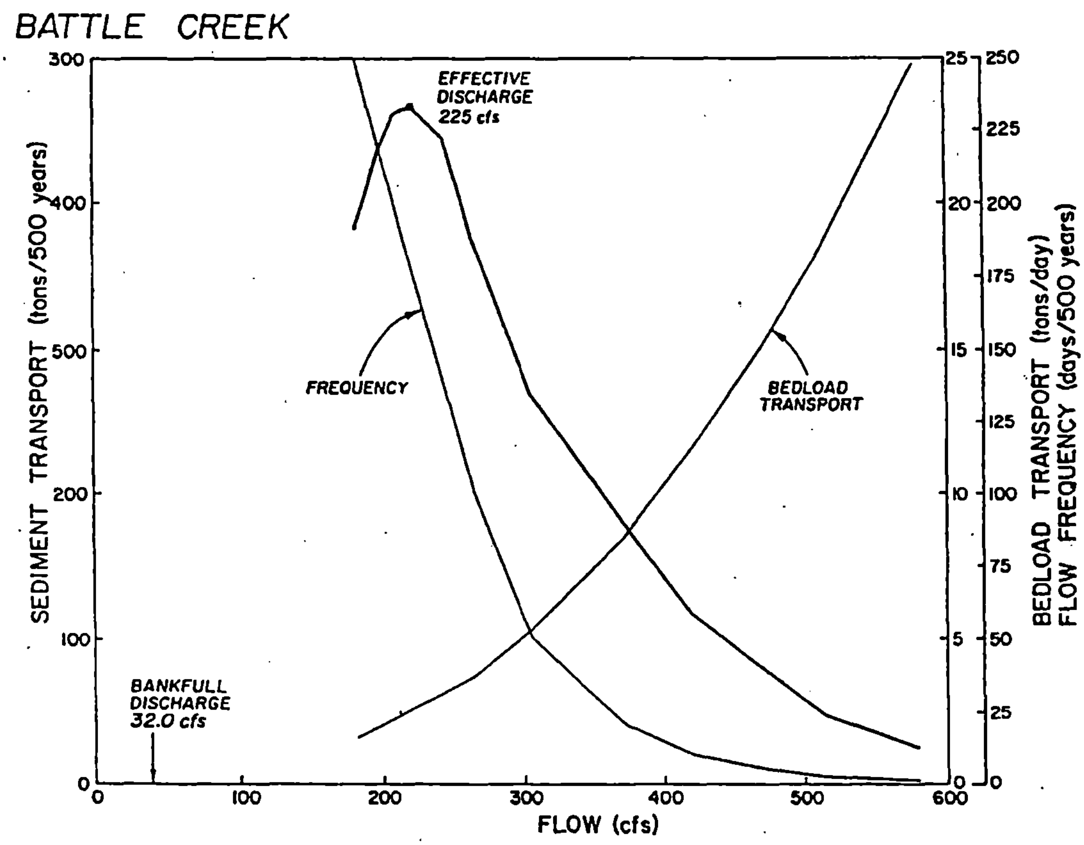

Figure 10. Magnitude-frequency analysis for Battle Creek.

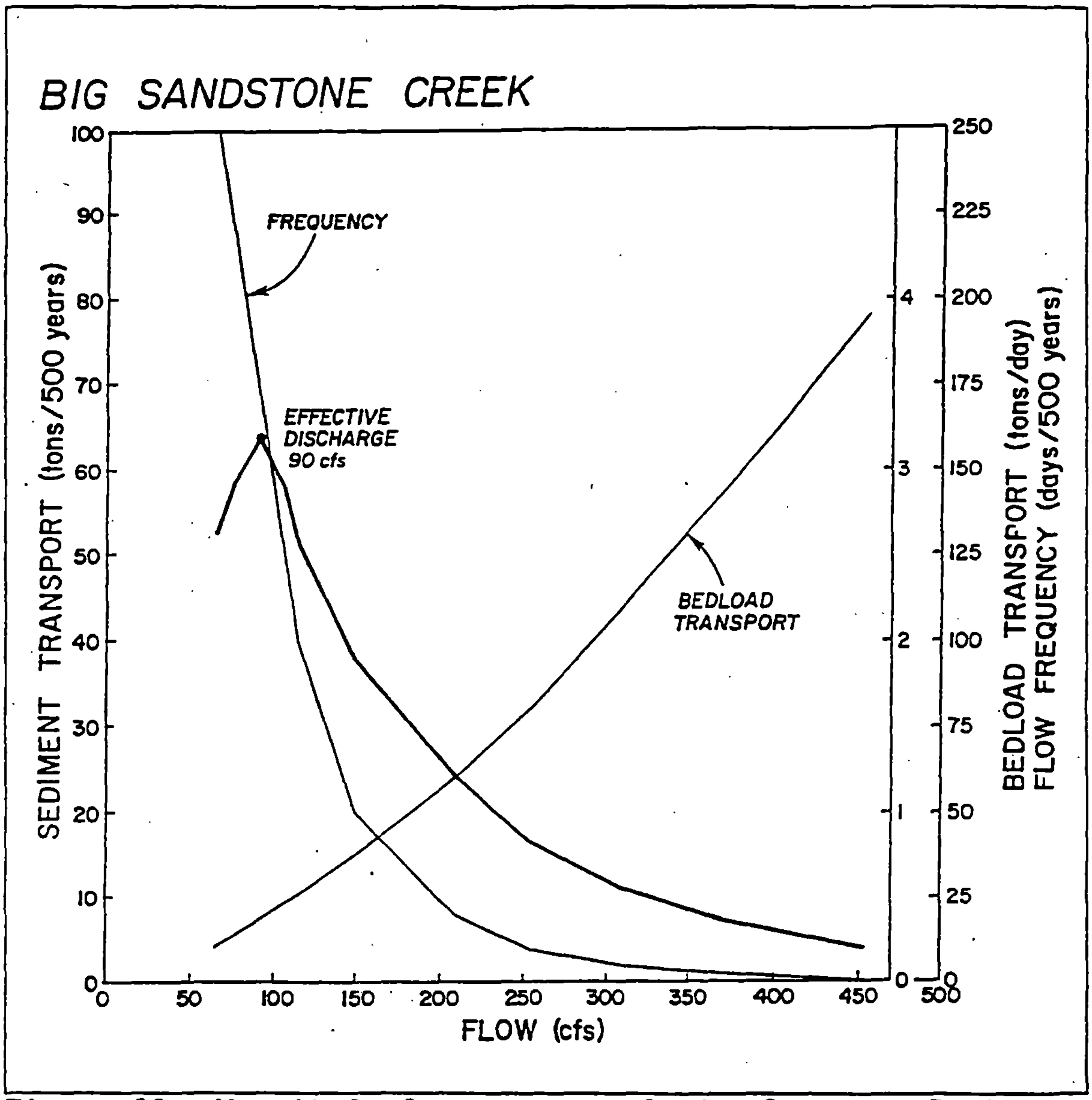

Figure 11. Magnitude-frequency analysis for Big Sandstone Creek.

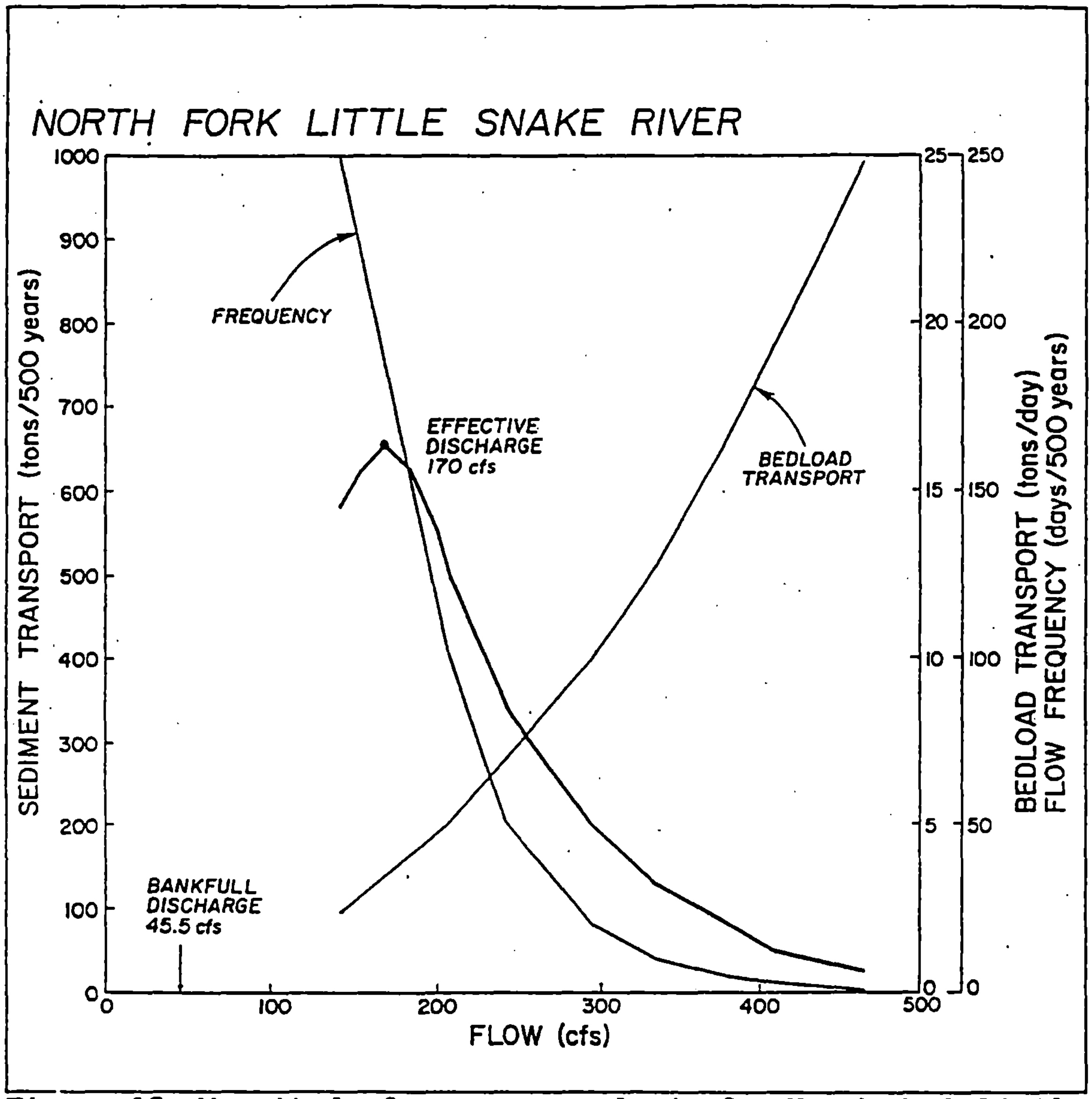

Figure 12. Magnitude-frequency analysis for North Fork Little Snake River.

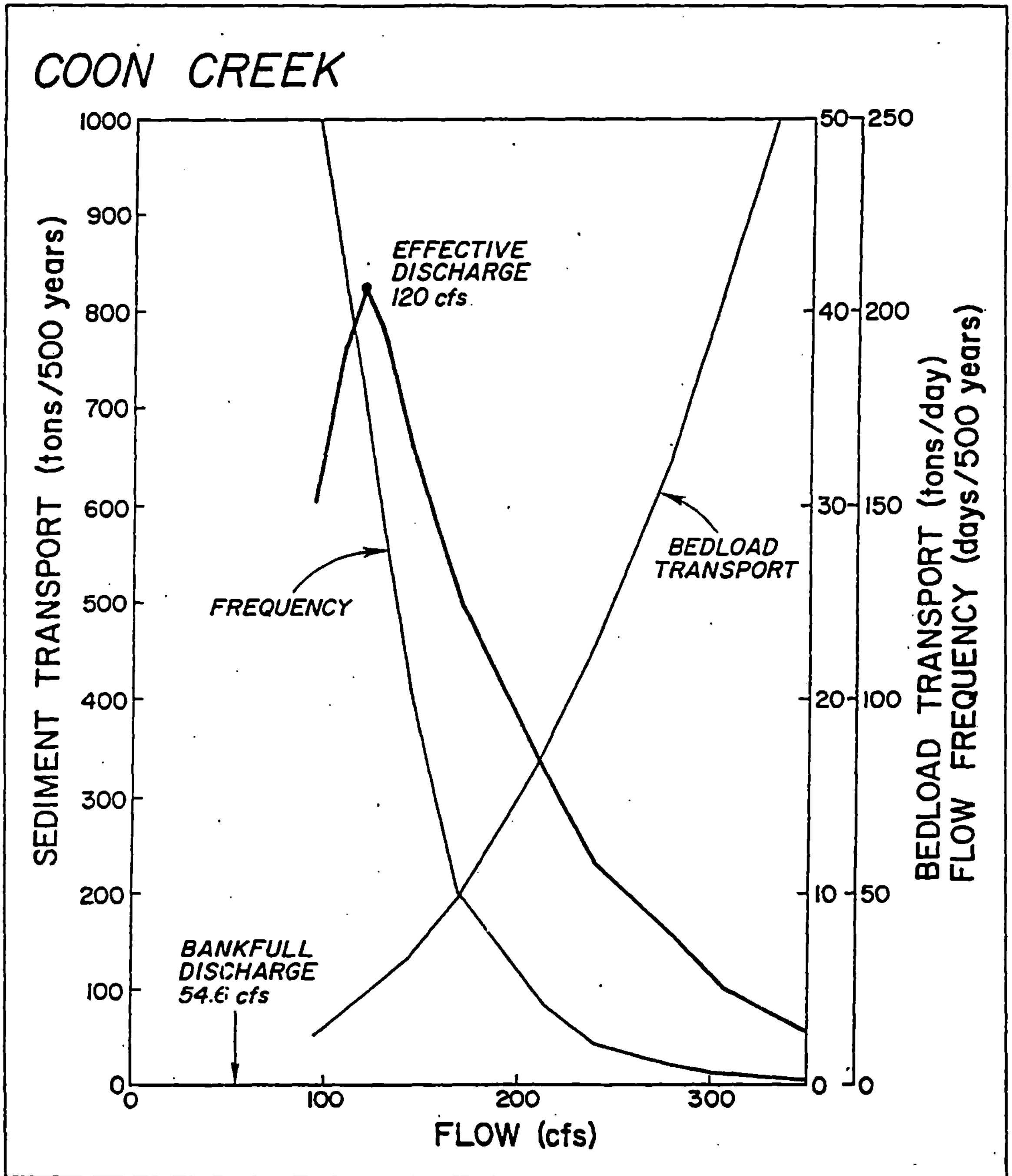

Figure 13. Magnitude-frequency analysis for Coon Creek.

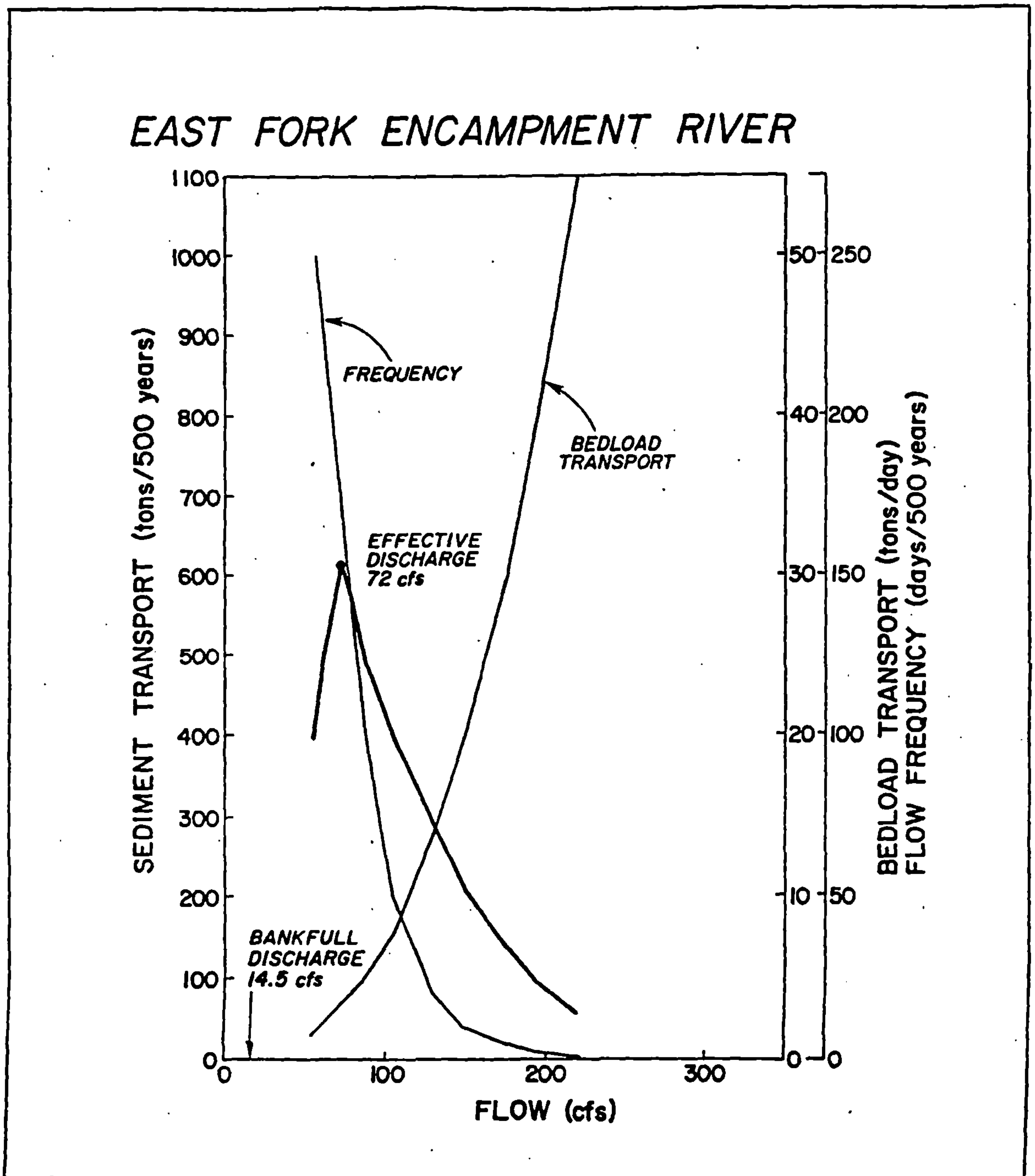

Figure 14. Magnitude-frequency analysis for East Fork Encampment River.

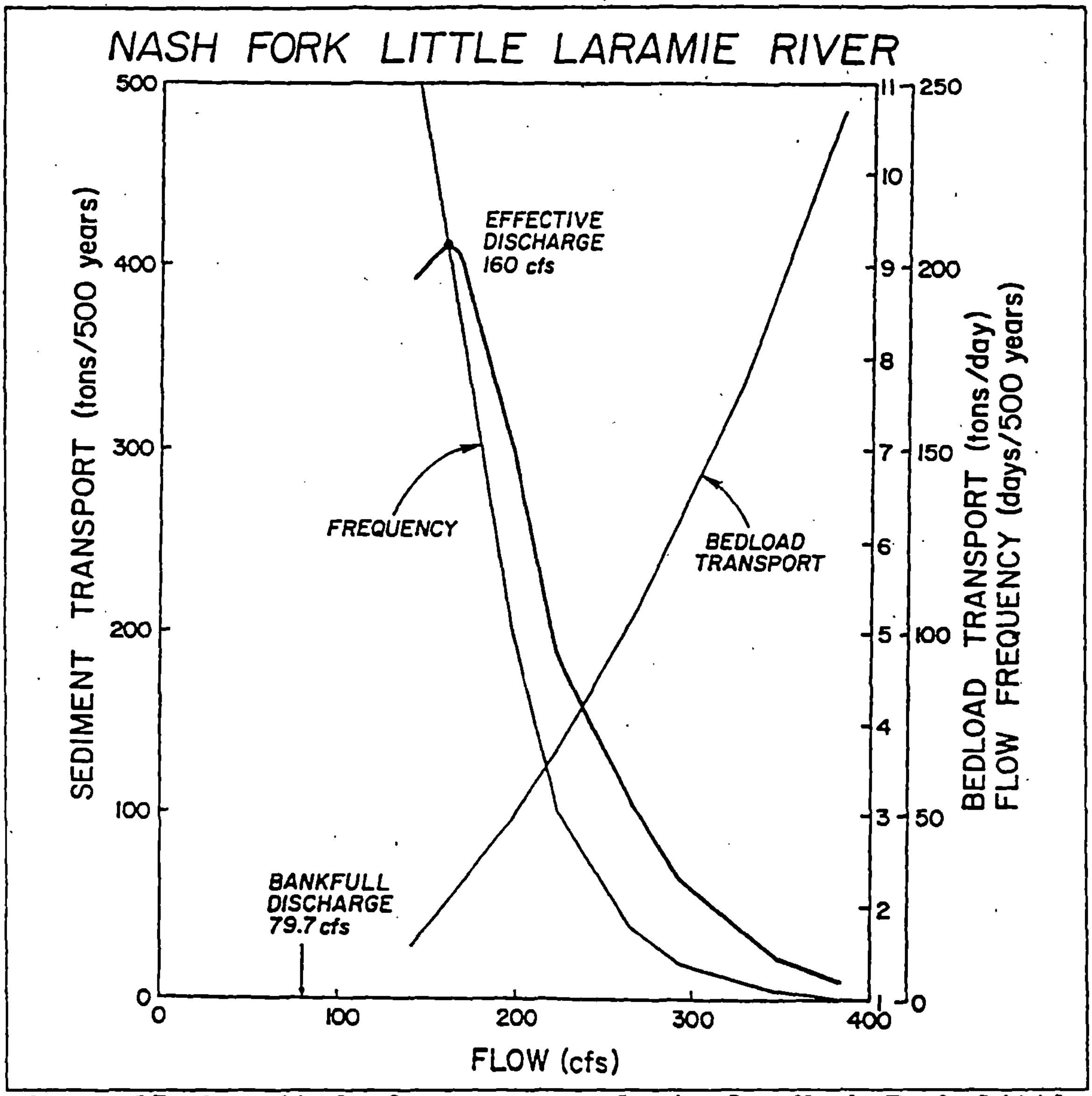

Figure 15. Magnitude-frequency analysis for Nash Fork Little Laramie River.

SUMMARY AND CONCLUSIONS

The approach of estimating suspended sediment or wash load with rating curves (using stream discharge) remains the best established method and the one most likely to improve. The development of a suite of rating curves for any given stream to account for the many confounding factors is an area of research which deserves attention.

Traditional formulae for estimating bed material transport have not proved useful in steep mountain streams because they were often developed in laboratory flumes with uniform bed substrate, spherical particles, small relative roughness, steady-uniform flow, low turbulence, particles which do not protrude from the channel bed, no bed topography, shallow sloping channels, no hydraulic jumps, fluctuating storage of sediment behind large woody debris, and unlimited sediment supply. In steep mountain streams, essentially all of these conditions are violated, leading to bed material transport rates which are apparently out-of-phase with hydraulics. Considering the confounding factors above, it is difficult to envision development of an accurate and universal deterministic model for predicting instantaneous measures of bed material transport. Separate sediment rating curves (whether they use discharge, shear stress, or unit stream power) should be developed for streams of contrasting morphology and substrate.

To develop separate sediment rating curves for streams of contrasting morphology and substrate, a stream reach classification scheme must be devised which accounts for the variation in morphology of mountain streams as it affects sediment transport. Following on the discussion above, this classification should address the mix of particle sizes in the substrate, measures of relative roughness, bed topography, channel planview pattern, channel gradient, channel cross-section shape, presence of hydraulic jumps, sediment storage effects of large woody debris, and whether the channel is supply-limited or transport-limited.

To link sediment transport relations with channel maintenance flows, the approach of Andrews is recommended. That is, estimates of effective discharge (determine from flow frequency curves and sediment rating curves) should be compared to field estimates of bankfull discharge. Channel maintenance flows are being met if the effective discharge--the flows transporting the most sediment--are contained in the channel at the bankfull stage. Research is needed to translate this measure of channel maintenance away from gaging stations to other points in the stream network.

Bed material transport is frequent in steep mountain streams of the Colorado-Wyoming Rocky Mountains. Data from the seven gaging stations of the Medicine Bow National Forest indicate that present channel morphology is not adjusted to the prevailing regime of water and sediment discharge. Increasing diversions of low

flows from transport-limited of the Medicine Bow National Forest would not greatly affect the inequality of effective discharge and bankfull discharge. Flows close to the effective discharge should be maintained to provide transport of sediment and maintain the channel.

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